

Name: Solutions - Section 031 PID: 42

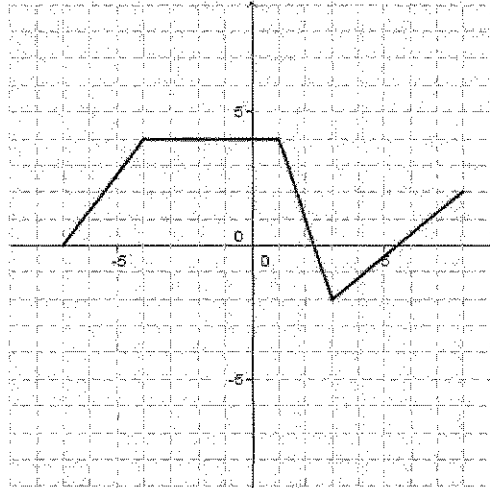
**Instructions:**

1. **DO NOT OPEN THIS EXAM UNTIL YOU ARE INSTRUCTED TO DO SO.**
2. Print your full name and your PID on your exam. Then finish reading these instructions, and **sign the bottom of this page.**
3. Without fully opening the exam, check the page numbers in this exam booklet. Including this cover page, you should have 7 different pages. If you do not, please request another copy immediately.
4. Neither books nor scratch paper are needed for this exam. Clear your desk of everything but this booklet, your pencils and your calculator. If you need more space to write your solutions, use the backs of the exam pages.
5. Calculators are not to be shared. Do not ask your instructor any questions about the use of your calculator. Only those calculators appropriate for MTH 124 (as specified in the course syllabus) are allowed for use during this exam.
6. All electronic equipment (such as cell phones, mp3 players, etc) must be turned off and stored away during the exam time.
7. Crib sheets (pre-compiled lists of formulas or other information) either written or in a calculator are specifically forbidden. **Use of a crib sheet of any kind on this exam will result in an automatic zero grade.**
8. **No talking is allowed during the exam.**
9. The problems on this exam vary in difficulty. You should try to solve these problems in an order that will maximize your score. Solve all the easier problems first, then go back to the ones that require more thought.
10. Unless otherwise indicated, **SHOW ALL YOUR WORK.** If no work is shown, no partial credit can be awarded. Even for calculator solutions, you should include relevant information, such as the equation to be solved, the function whose graph is to be sketched, etc. Your work and answer need to be accurate and relevant to receive points.
11. Unless you are specifically instructed to do otherwise, **DO NOT ROUND YOUR ANSWERS – GIVE EXACT ANSWERS.**
12. You will be given **exactly 50 minutes** for this exam.
13. *Any student not following the above instructions nor behaving according to the above instructions during the exam may have their exam confiscated and points deducted.*

I have read and fully understood all of the above instructions: **Signature:** \_\_\_\_\_

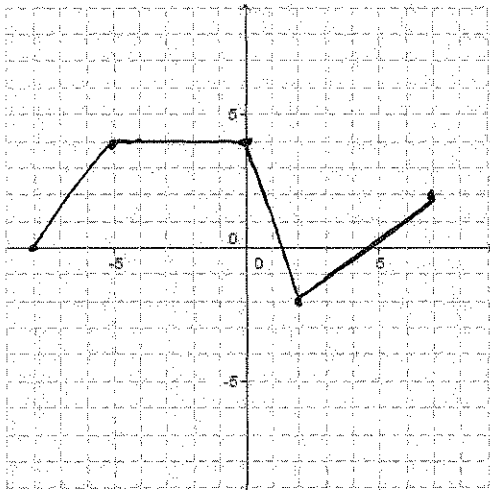
Problem	1	2	3	4	5	6	7	8	9	TOTAL
Points										
Maximum	13	9	12	12	10	12	10	14	8	100

1. The graph of a function,  $f(x)$ , is given below.  $[9 + 4 = 13]$

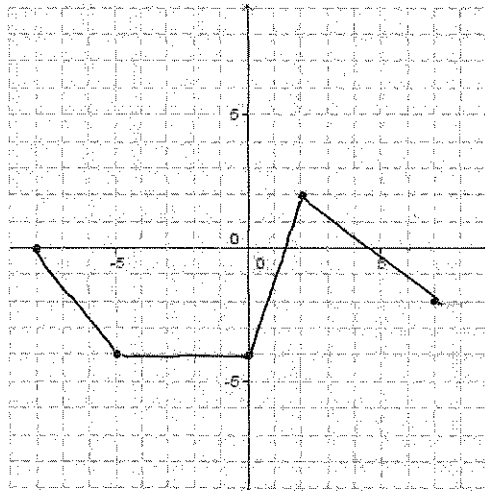


(a) First, draw the graph of  $f(x + 1)$ . Then, draw the graph of  $-f(x + 1)$ .

Left 1 unit

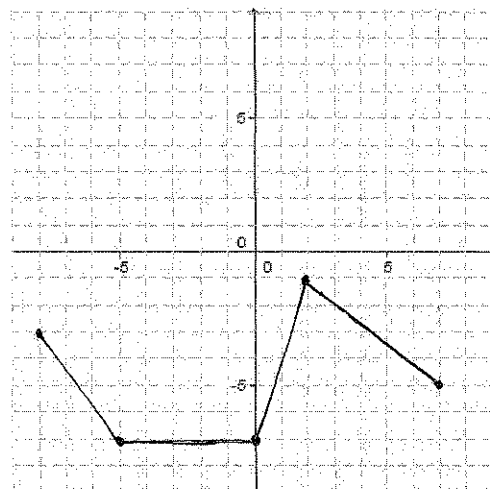


Flip vertically



(b) Finally, draw the graph of  $-f(x + 1) - 3$ .

Down 3.



2. If  $f(x) = \sqrt{x+5}$  and  $g(x) = 2x^2 - 3x$ , then what is  $g(f(g(4)))$ ? [9]

$$g(4) = 2 \cdot 4^2 - 3 \cdot 4 = 2 \cdot 16 - 12 = \underline{20}$$

$$f(20) = \sqrt{20+5} = \sqrt{25} = \underline{5}$$

$$g(5) = 2 \cdot 5^2 - 3 \cdot 5 = 2 \cdot 25 - 15 = \underline{35}$$

$$\text{So } g(f(g(4))) = g(f(20)) = g(5) = \boxed{35}$$

3. (a) Let  $z$  be directly proportional to the square root of  $x$ . If  $z = 2$  when  $x = 16$ , what is  $x$  when  $z = 7$ ? [6]

$$z = k\sqrt{x}$$

$$z = \frac{1}{2}\sqrt{x}$$

• Find  $k$ :  $z = k\sqrt{16} = 4k \longrightarrow k = \frac{2}{4} = \frac{1}{2}$

$$z = 7: 7 = \frac{1}{2}\sqrt{x} \longrightarrow 14 = \sqrt{x} \longrightarrow x = 14^2$$

$$\boxed{x = 196}$$

(b) Let  $x$  be inversely proportional to  $y^3$ . If  $x = 6$  when  $y = 2$ , what is  $x$  when  $y = 4$ ? [6]

$$x = \frac{k}{y^3}$$

$$x = \frac{48}{y^3}$$

• Find  $k$ :  $6 = \frac{k}{2^3} = \frac{k}{8} \longrightarrow k = 6 \cdot 8 = 48$

$$y = 4: x = \frac{48}{4^3} = \frac{48}{64} = \frac{3}{4}$$

$$\boxed{x = \frac{3}{4} = 0.75}$$

4. Using the definition of derivative, find  $f'(x)$  when  $f(x) = 3x^2$ . [12]

$$\begin{aligned} f(x+h) &= 3(x+h)^2 = 3(x^2 + 2hx + h^2) \\ &= 3x^2 + 6hx + 3h^2 \end{aligned}$$

$$\begin{aligned} \frac{f(x+h) - f(x)}{h} &= \frac{\cancel{3x^2} + 6hx + 3h^2 - \cancel{3x^2}}{h} = \frac{6hx + 3h^2}{h} \\ &= 6x + 3h \end{aligned}$$

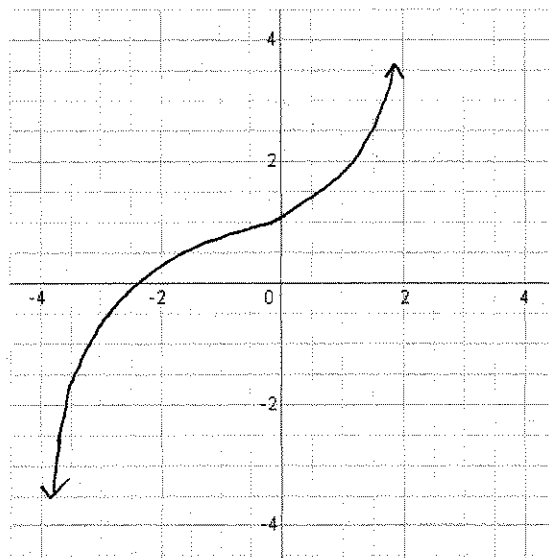
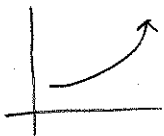
$$\text{So } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$f'(x) = 6x$

$$= \lim_{h \rightarrow 0} [6x + 3h] = 6x + 0$$

5. Sketch the graph of a function  $f$  such that  $f'(x) > 0$  for all  $x$ ,  $f''(x) < 0$  for  $x < 0$ , and  $f''(x) > 0$  for  $x > 0$ . [10]

Concave up:



Increasing

Concave down:



6. In a time of  $t$  seconds, a particle moves a distance of  $s$  meters from its starting point, where  $s = t^3$ . [9 + 3 = 12]

(a) Find the average velocity between  $t = 1$  and  $t = 1 + h$  for the given values of  $h$  below. Round your answers to three decimal places.

i.  $h = 0.1$

$$v = s'(t) \approx \frac{\Delta s}{\Delta t}$$

$$\frac{s(1+h) - s(1)}{h} = \frac{s(1.1) - s(1)}{0.1} = \frac{(1.1)^3 - 1^3}{0.1} \quad \text{Units: meters/sec.}$$

$$= \frac{1.331 - 1}{0.1} = \boxed{3.310 \text{ m/s}}$$

ii.  $h = 0.01$

$$\frac{s(1.01) - s(1)}{0.01} = \frac{(1.01)^3 - 1^3}{0.01} = \frac{1.030301 - 1^3}{0.01}$$

$$\approx \boxed{3.030 \text{ m/s}}$$

iii.  $h = 0.001$

$$\frac{s(1.001) - s(1)}{0.001} = \frac{(1.001)^3 - 1^3}{0.001}$$

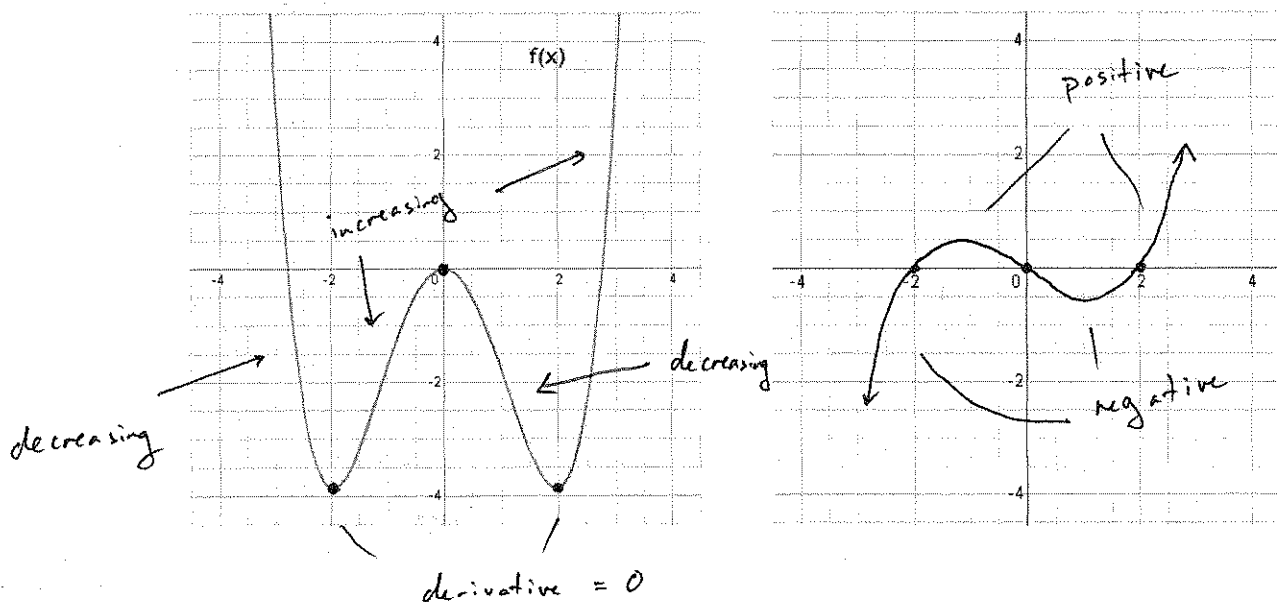
$$\approx \boxed{3.003 \text{ m/s}}$$

(b) Use your answers to part (a) to estimate the instantaneous velocity of the particle at time  $t = 1$ . Explain your answer.

The values seem to be getting arbitrarily close to 3 as  $h$  goes to 0.

So we estimate  $\boxed{v(1) = 3 \text{ m/s}}$ .

7. Using the axes below, sketch a graph of the derivative of  $f(x)$ . [10]



8. The quantity,  $Q$  in mg, of nicotine in the body  $t$  minutes after a cigarette is smoked is given by  $Q = f(t)$ . [8 + 6 = 14]

(a) Interpret the statements  $f(20) = 0.36$  and  $f'(20) = -0.002$  in terms of nicotine. What are the units of the numbers 20, 0.36 and  $-0.002$ ?

After 20 minutes, there are 0.36 mg of nicotine in the body. This is decreasing at a rate of 0.002 mg per minute.

Units:  $\begin{matrix} 20 & \text{minutes} \\ 0.36 & \text{mg nicotine} \\ -0.002 & \text{mg nicotine per minute} \end{matrix}$   $\downarrow$  = mg/min.

(b) Use the information given in part (a) to estimate  $f(21)$  and  $f(30)$ . Justify your answers.

Using  $f' \approx \frac{\Delta y}{\Delta x}$ :

• Between 20 & 21,  $\Delta x = 1$  so

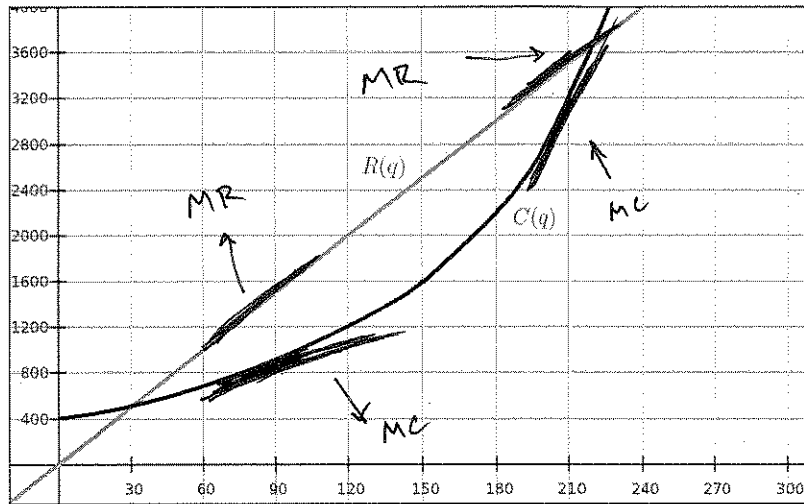
$$\Delta y \approx 1 f'(20) = -0.002.$$

$$\text{So } f(21) \approx f(20) - 0.002 = \boxed{0.358 \text{ mg}}$$

• Between 20 & 30,  $\Delta x = 10$ .  $\Delta y \approx 10 f'(20) = -0.02$ .

$$\text{So } \boxed{f(30) \approx 0.34 \text{ mg}}$$

9. Cost and Revenue functions  $C(q)$  and  $R(q)$  correspondingly for the company that produces heaters is shown in the figure below.  $[4 + 4 = 8]$



Marginal cost:  
 $MC = C'$

Marginal revenue  
 $MR = R'$

- (a) Is it a good idea to increase the production level from 90 to 91 heaters? Explain using marginal revenue and marginal cost.

Slope of  $C$  is less than slope of  $R$

Yes

→ Marginal cost < Marginal revenue.

→ We want to increase production, since the next item brings in more revenue than it costs to produce.

- (b) Is it a good idea to increase the production level from 210 to 211 heaters? Explain using marginal revenue and marginal cost.

Marginal cost exceeds marginal revenue, so

No

no increase in production.

